Deterministic Finite Automata

Arjun Chandrasekhar

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  - Some states are labelled as *accept states*
- Transitions (arrows) between states
  - Each transition is labelled by a character from the alphabet

Deterministic Finite Automata

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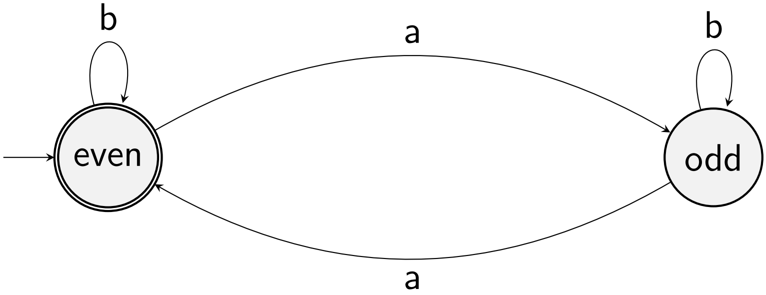

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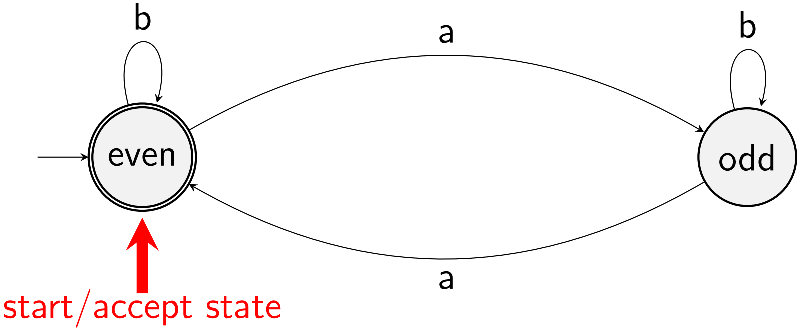

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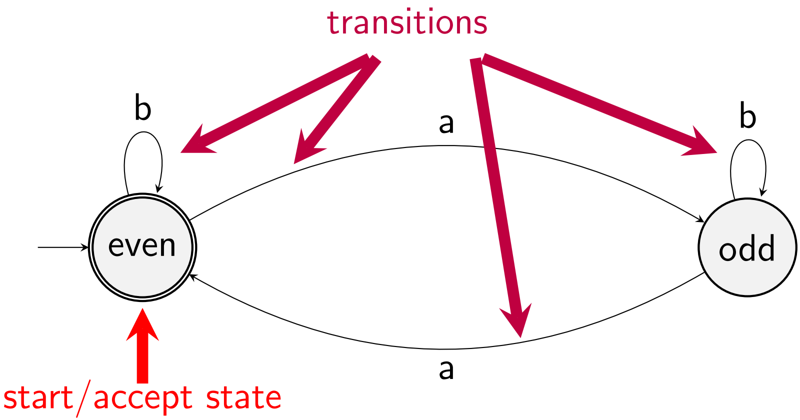

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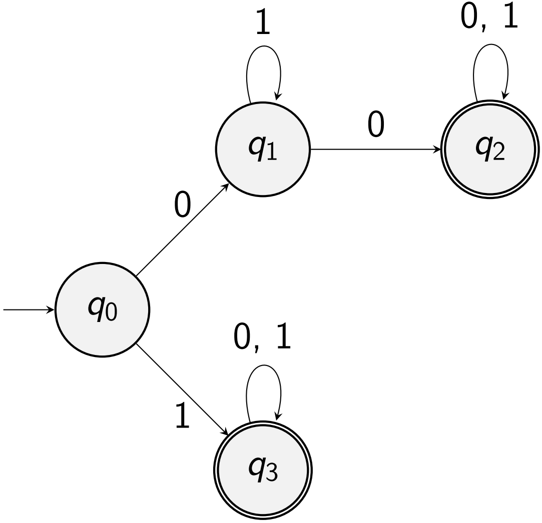

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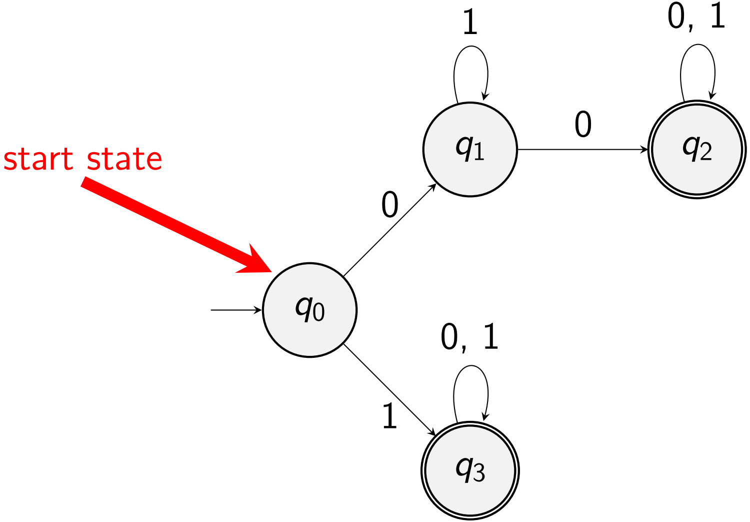

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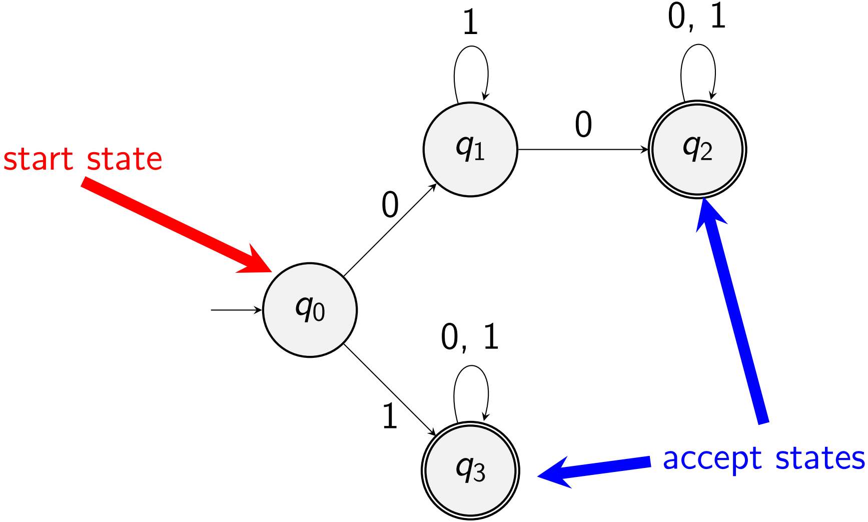

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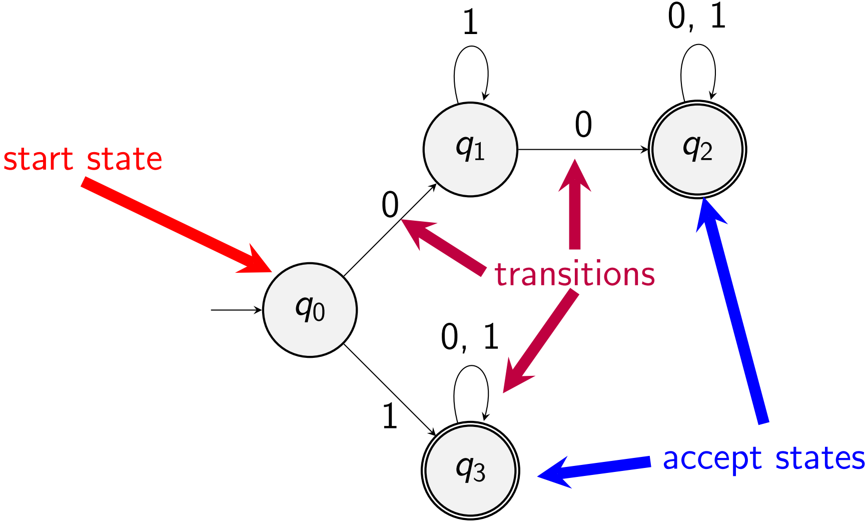

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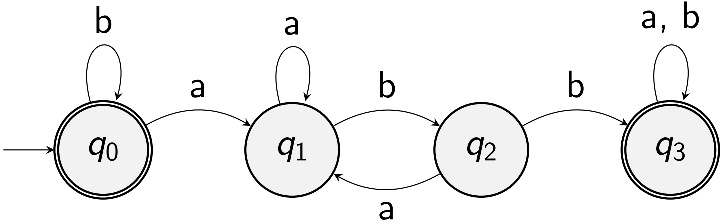


What is the start state of this DFA?

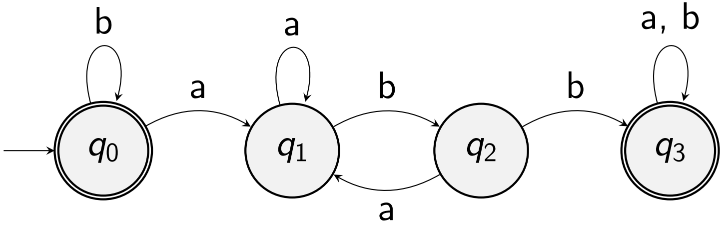
A. q_0

B. q_1

C. q_2

D. q_3

Deterministic finite automata



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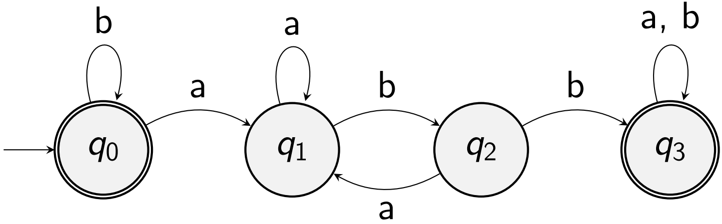

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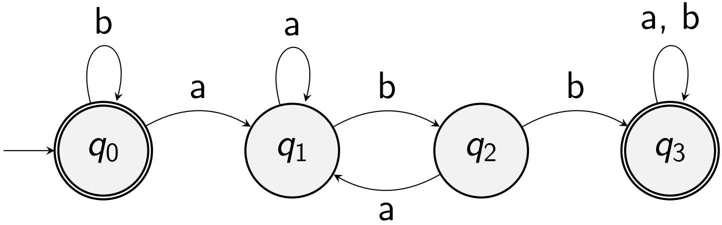

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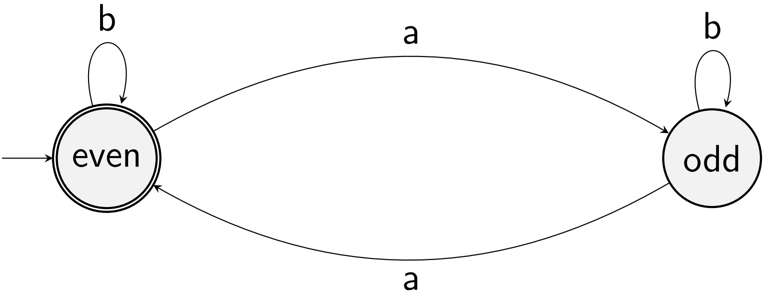
Computation on a DFA

- Start in the start state
- Scan the symbols in the input one by one
- For each symbol σ scaned:
  - Go to the next state by following the arrow with the label σ
- After scanning all of the input, if the DFA is in an accept state, the input is accepted. Otherwise the input is rejected

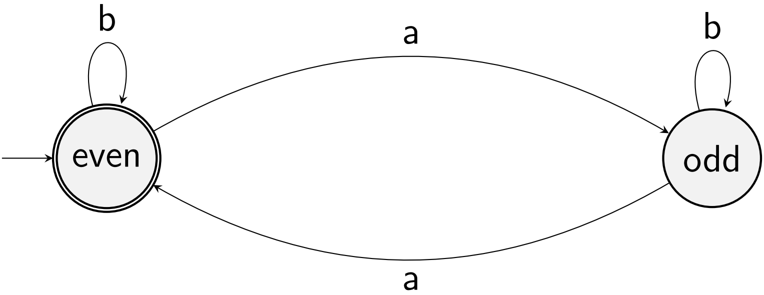
Deterministic finite automata

What does this DFA do on input aaaa?



Deterministic finite automata

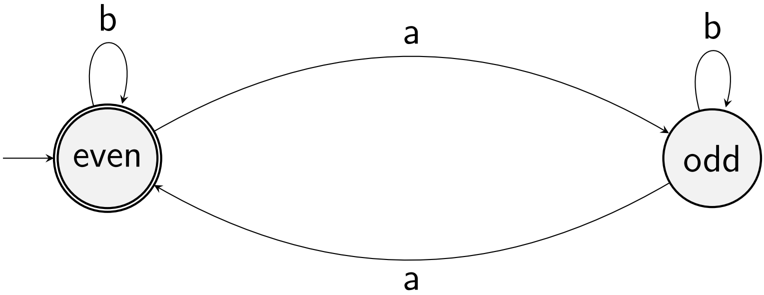
What does this DFA do on input aaaa?



$$\text{even} \xrightarrow{a} \text{odd} \xrightarrow{a} \text{even} \xrightarrow{a} \text{odd} \xrightarrow{a} \text{even} \rightarrow \text{ACCEPT}$$

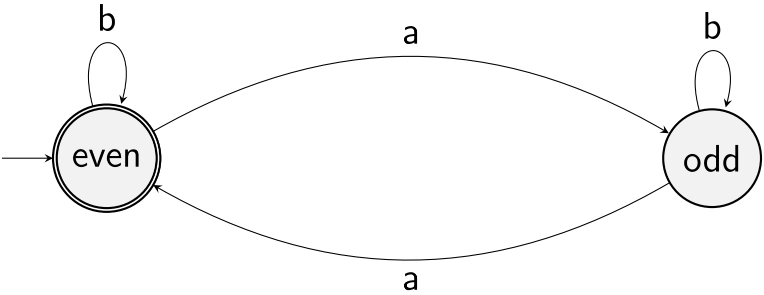
Deterministic finite automata

What does this DFA do on input b?



Deterministic finite automata

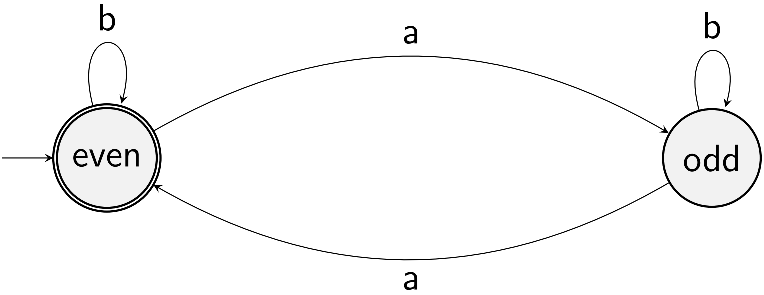
What does this DFA do on input b?



$$\text{even} \xrightarrow{b} \text{even} \rightarrow \text{ACCEPT}$$

Deterministic finite automata

What does this DFA do on input abb?



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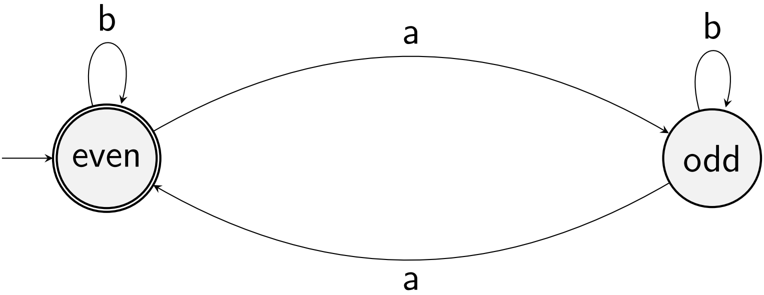
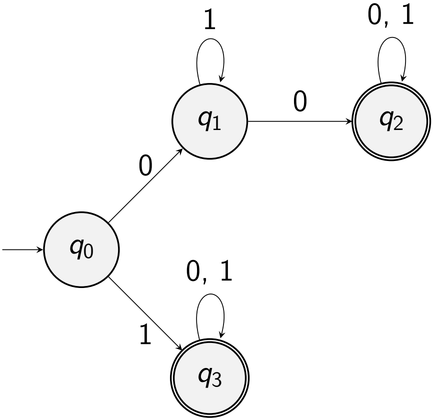


$$\text{even} \xrightarrow{a} \text{odd} \xrightarrow{b} \text{odd} \xrightarrow{b} \text{odd} \rightarrow \textsf{REJECT}$$

Deterministic Finite Automata

What does this DFA do on input 0110?



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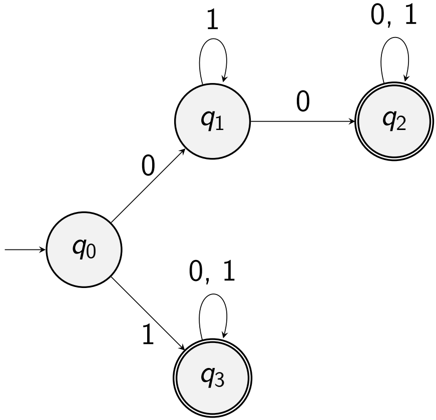
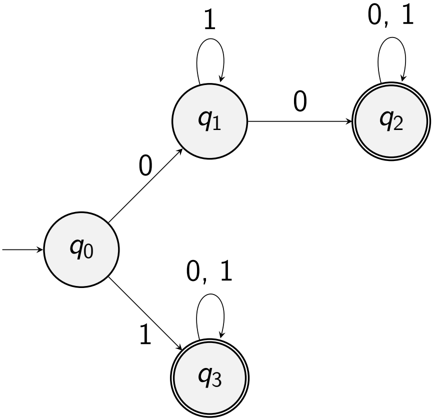


$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \rightarrow \text{ACCEPT}$$

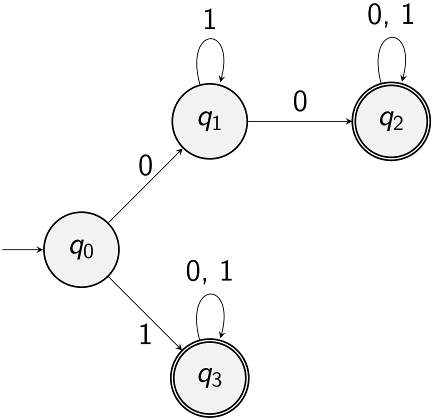
Deterministic Finite Automata

What does this DFA do on input 1011?



Deterministic Finite Automata

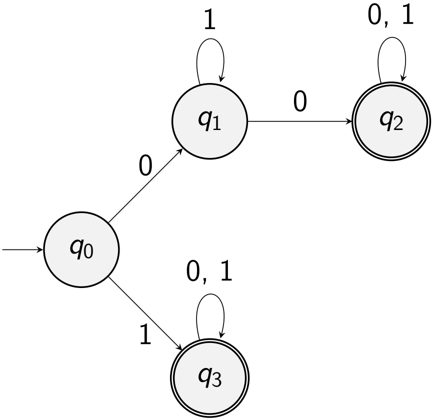
What does this DFA do on input 1011?



$$q_0 \xrightarrow{1} q_3 \xrightarrow{0} q_3 \xrightarrow{1} q_3 \xrightarrow{1} q_3 \rightarrow \text{ACCEPT}$$

Deterministic Finite Automata

What does this DFA do on input 0111?



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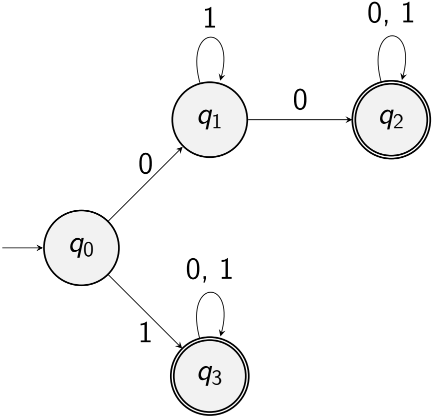
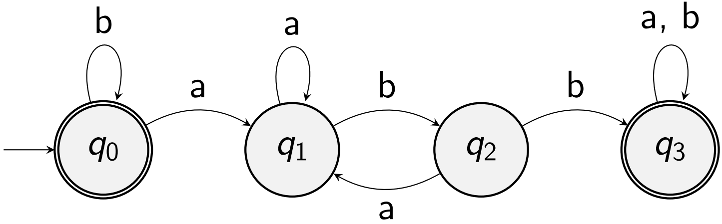


$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \xrightarrow{1} q_1 \rightarrow \text{REJECT}$$

Deterministic Finite Automata



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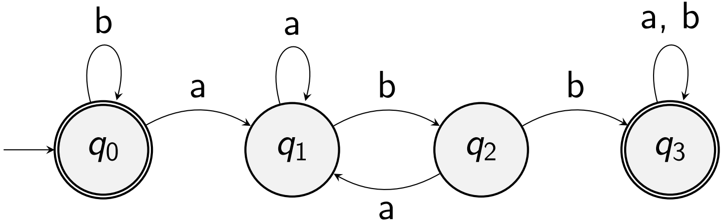

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- F - the set of accepting states

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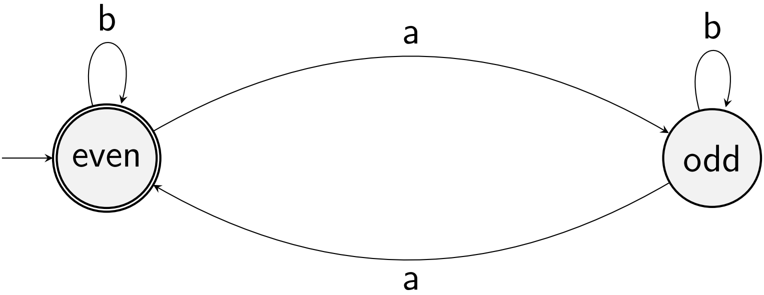
- $q_i = \delta(q_{i-1}, \sigma_i)$ for all i (all transitions are valid)
- $q_n \in F$ (finish in an accept state)

Formal definition example



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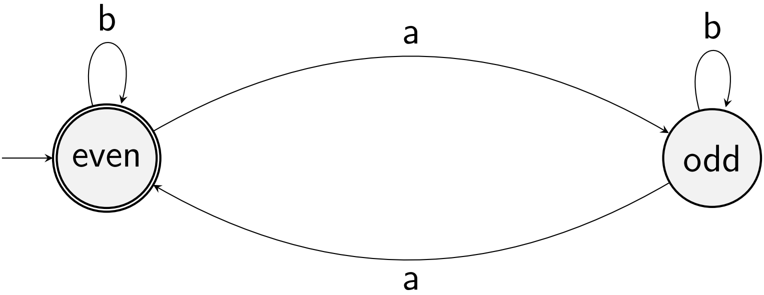

- $Q = \{\text{even}, \text{odd}\}$

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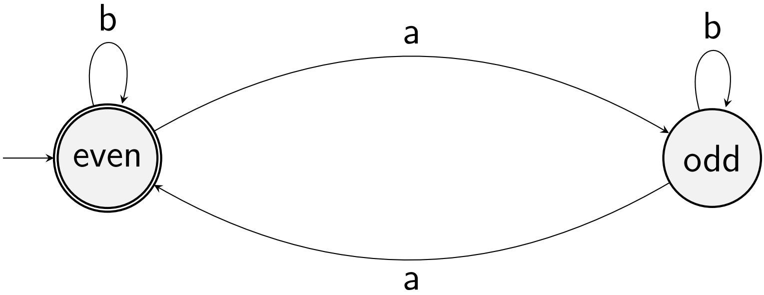

- Q = {even, odd}
- $\Sigma = \{a, b\}$

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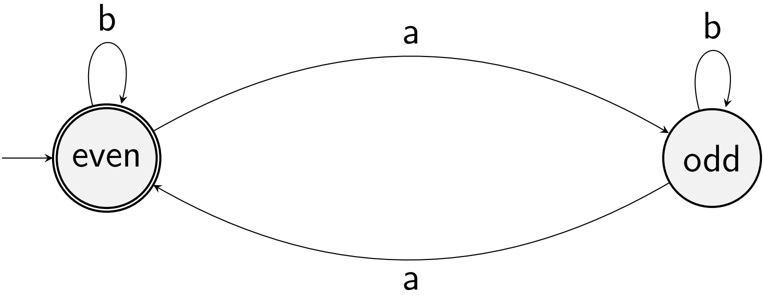

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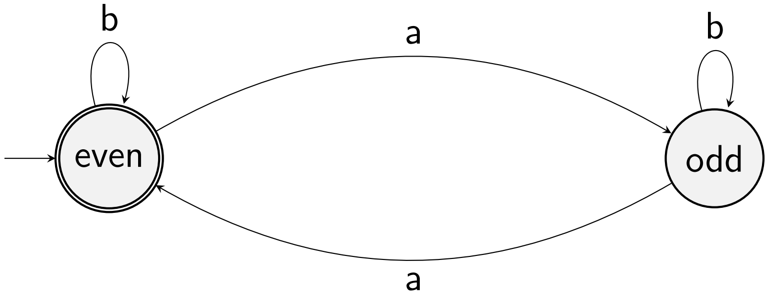

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- $q_s = \text{even}$
- $F = \{\text{even}\}$

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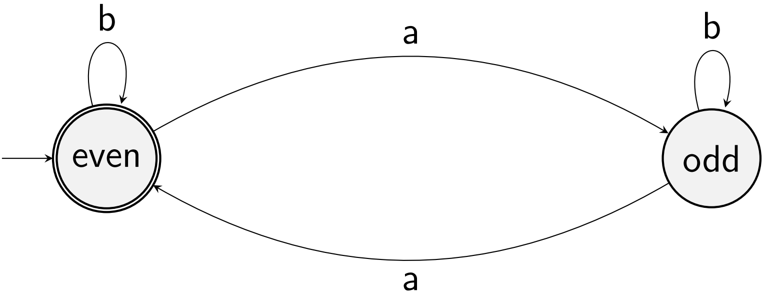

- $Q = \{\text{even, odd}\}$
- $\Sigma = \{a, b\}$
- $q_s = \text{even}$
- $F = \{\text{even}\}$

- $\delta =$

	a	b

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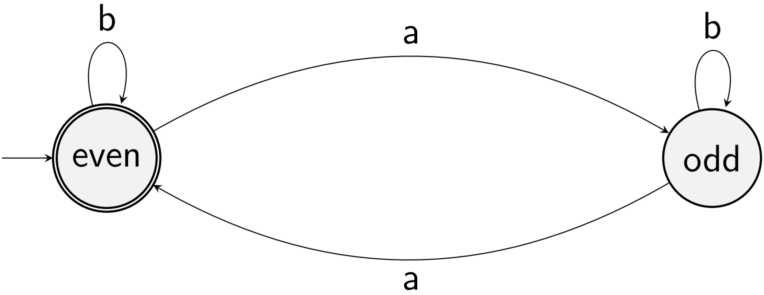

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even	odd	even

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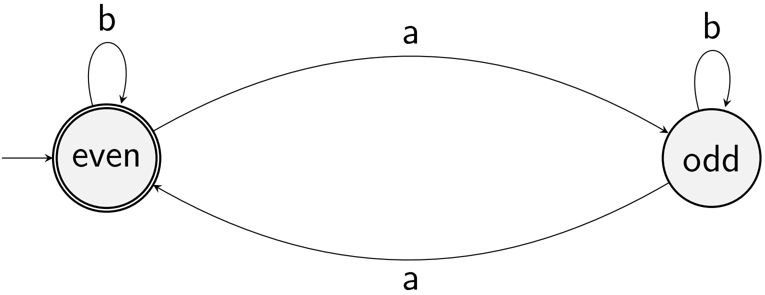

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	a	b
even	odd	even
odd	even	odd

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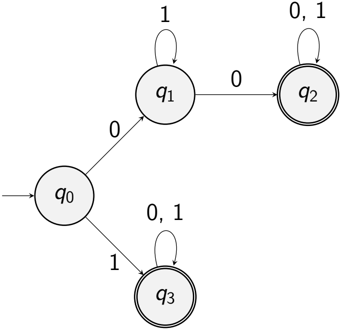

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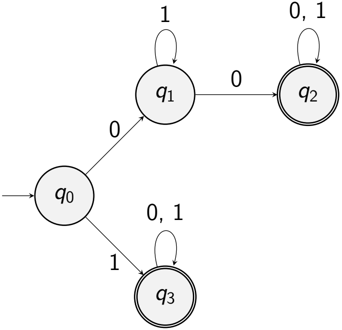

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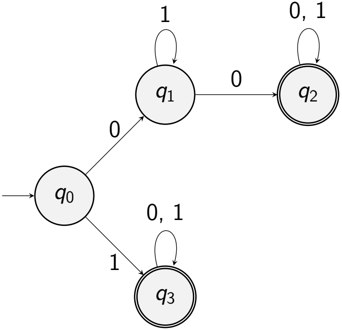

- $Q = \{q_0, q_1, q_2, q_3\}$
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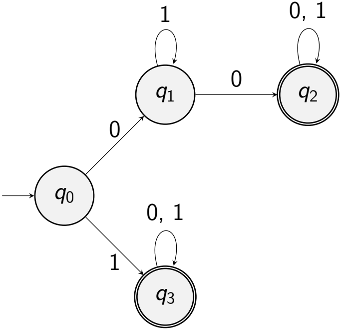

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- $q_s = q_0$

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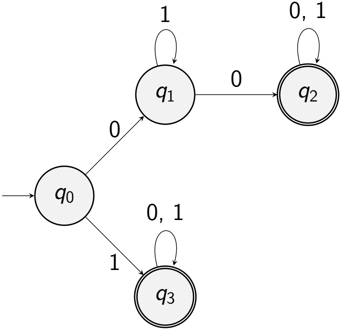

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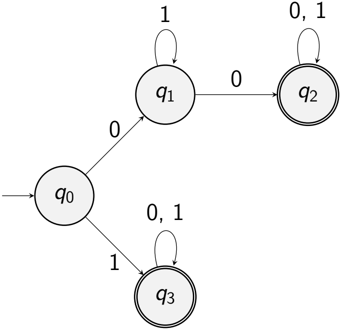

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- $\delta =$

	0	1

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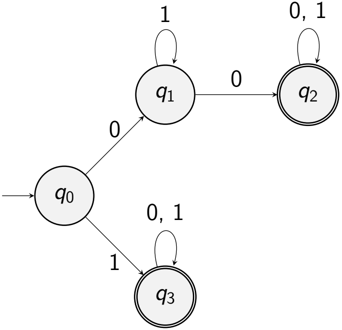

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q_0	q_1	q_3

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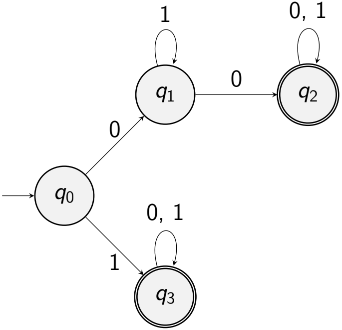

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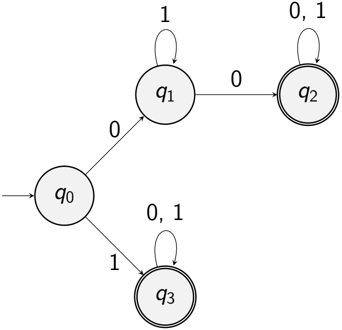

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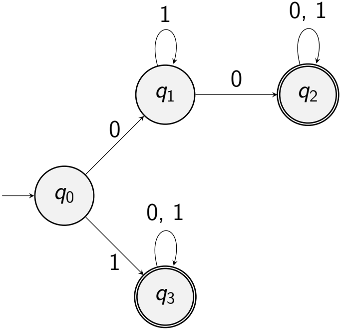
q_0	q_1	q_3
q_1	q_2	q_1
q_2	q_2	q_2

Write the formal definition



- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{0, 1\}$
- $q_s = q_0$
- $F = \{q_2, q_3\}$
- $\delta =$

	0	1
q_0	q_1	q_3
q_1	q_2	q_1
q_2	q_2	q_2
q_3	q_3	q_3

The language of a DFA

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  - If $w \in L$, then D accepts w
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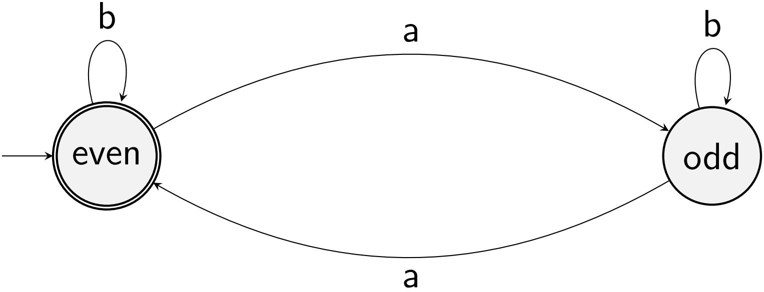
  - Do we ever have to worry about D looping?
- The **language of D**, denoted $L(D)$ is the (unique) language that D recognizes - that is, the set of all strings that D accepts

The language of a DFA

What the language of this DFA?



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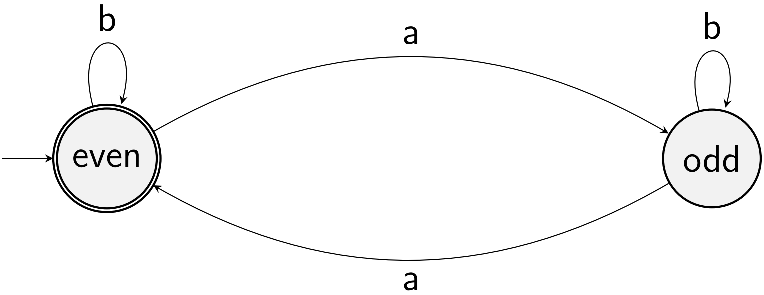

$L(D) = \{\text{w} \mid \text{w has an even number of a's}\}$

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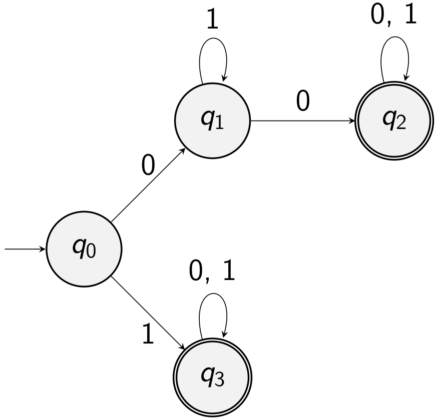

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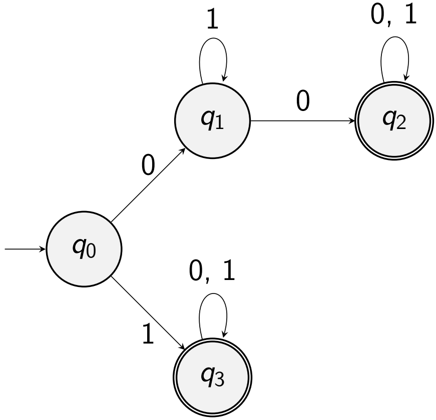

$L(D) = \{$w | w starts with 1 or has at least two 0's$\}$

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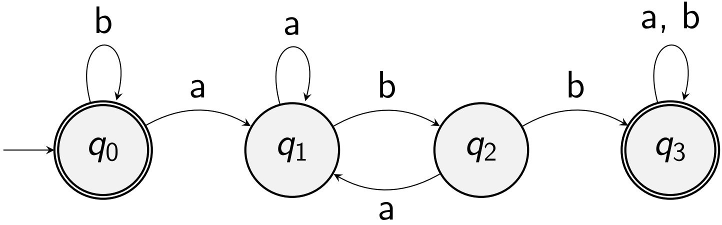

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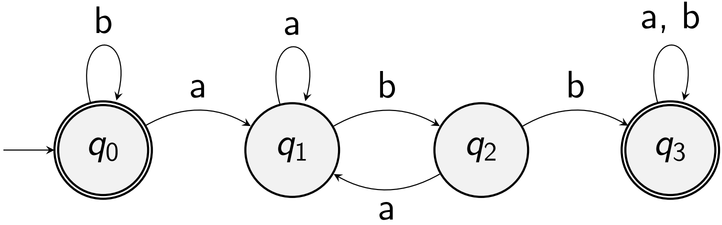

$L(Q) = \{\text{w} \mid \text{w contains only b's or contains abb}\}$

Deterministic Finite Automata

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- There can be more than one accepting state.
- The set of languages that can be recognized by some DFA is called the **regular languages**.
    - A language L is regular if and only if some DFA D recognizes L

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Let $\Sigma = \{a, b\}$. Let's show that the following language is regular:

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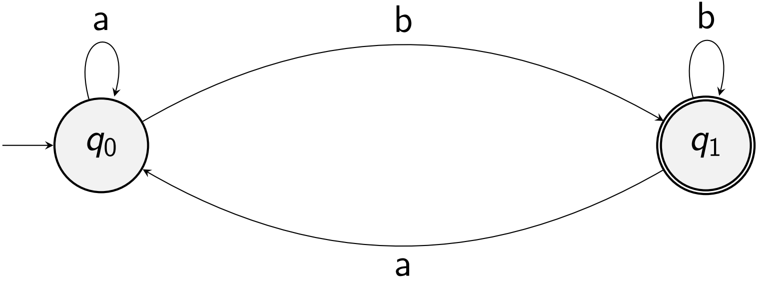

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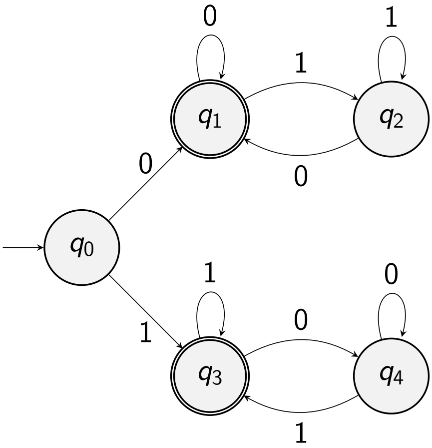

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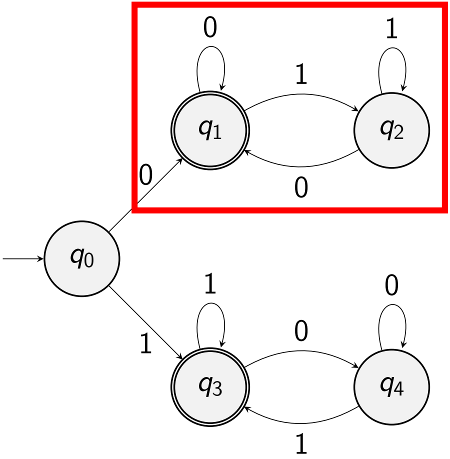

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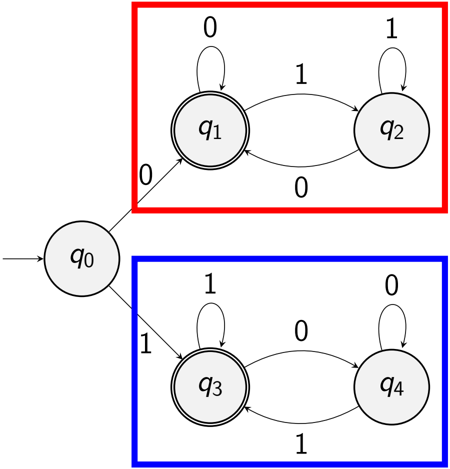

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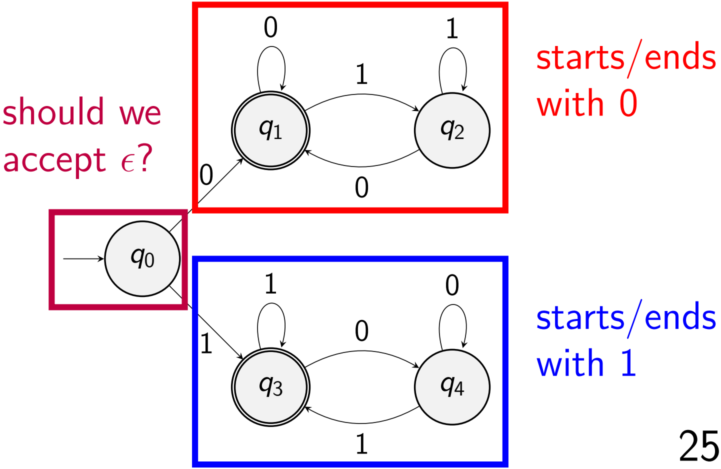

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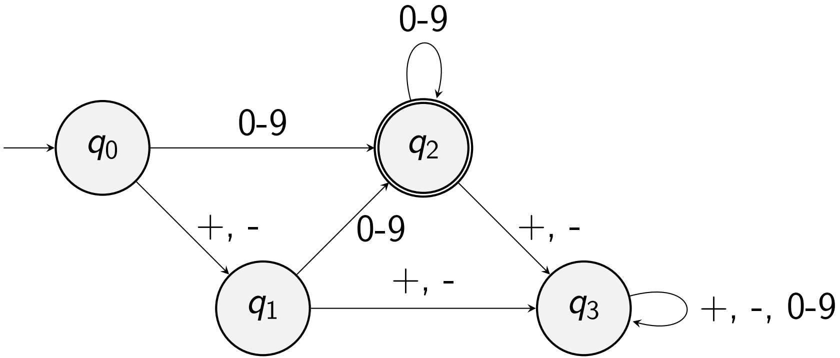

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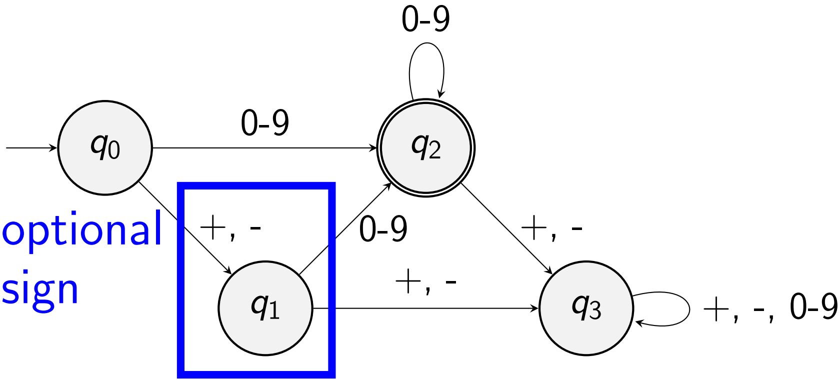

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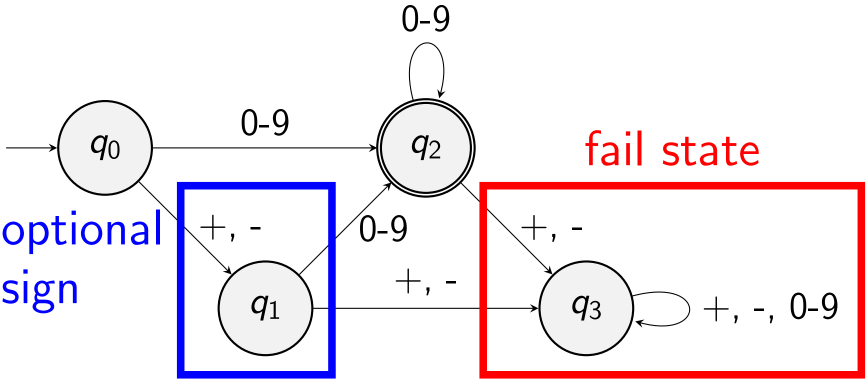

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