Regular Expressions

Arjun Chandrasekhar

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- Another way to describe languages

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- A formula that can be used to generate strings

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- A formula that can be used to generate strings
- Formed by combining smaller regular expressions using the three regular operations

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6. R_1^* where R_1 is a regular expression

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Empty set is different from empty string!

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- R^+ is shorthand for RR^* (which is shorthand for $R \circ R^*$)

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- String search tools that are built into operating system utilities (like awk and grep in Unix), text editors, and programming language libraries. Regular expressions describe the strings that are being searched for.
- The regular expressions in these tools typically have a richer set of operators, to facilitate more easily describing strings.

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1. $\Sigma^* = (0 \cup 1)^*$ = all binary strings (you have already seen this one!)
2. $\Sigma^* 001 \Sigma^* = \{w \mid w \text{ contains } 001 \text{ as a substring}\}$
3. $0 \cup 1 \cup (0\Sigma^*0) \cup (1\Sigma^*1) = \{w \mid w \text{ starts and ends with the same symbol}\}$

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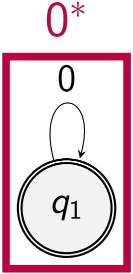

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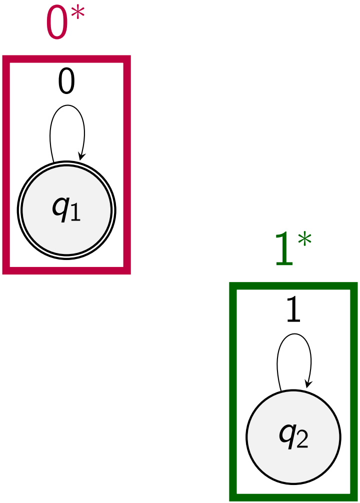

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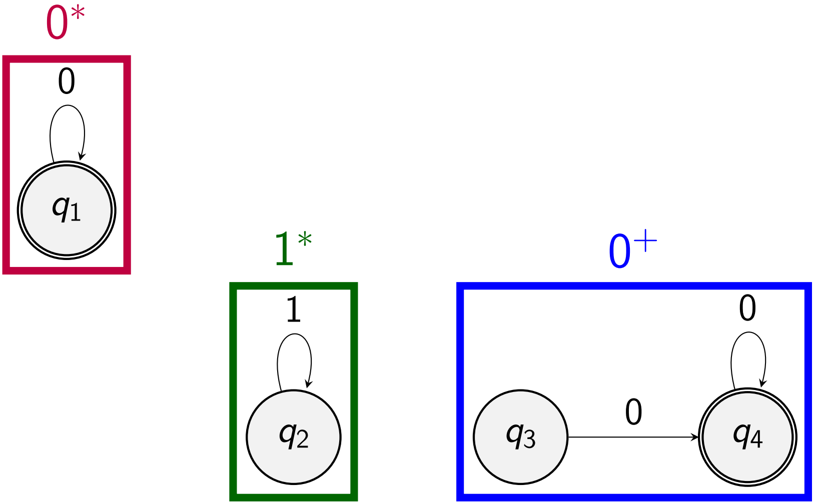

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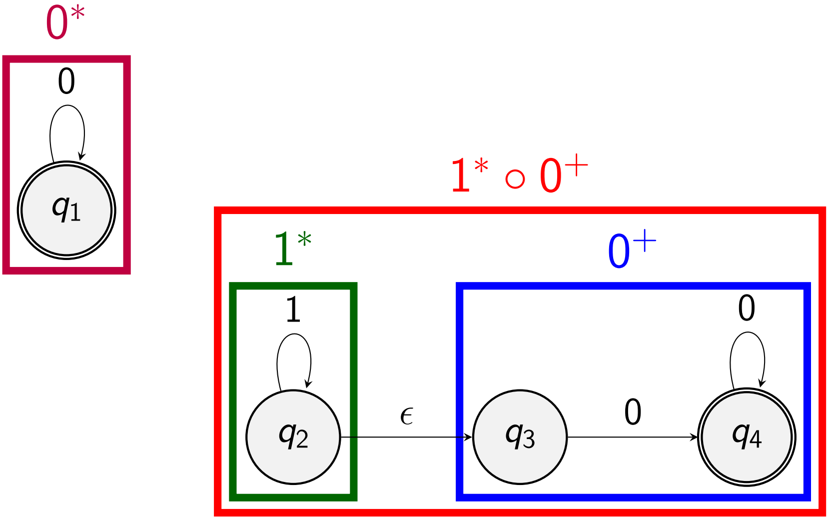

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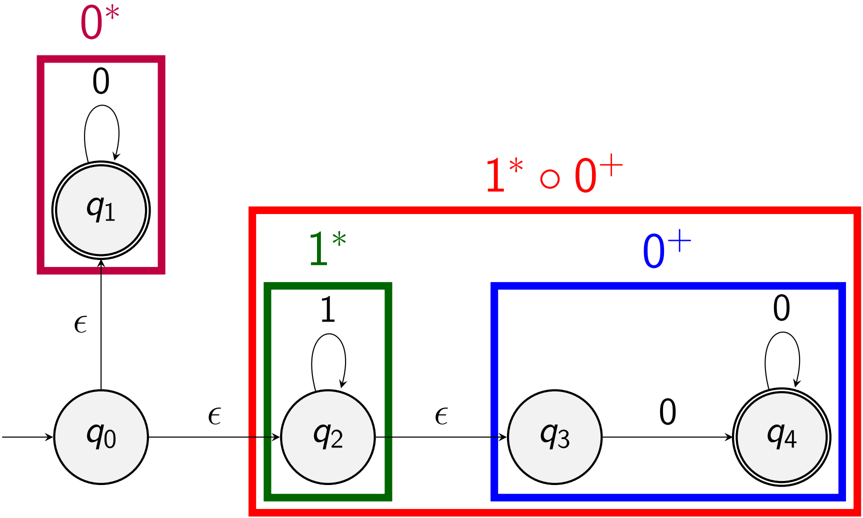

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  - ($\Leftarrow$) If a language is regular, then it is described by a regular expression
- **Recall:** A language is regular if and only if it is described by a DFA
  - Or equivalently (and conveniently), an NFA

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- For union, concatenation, and star, use induction to make NFAs for the smaller parts of the expression, and then combine them

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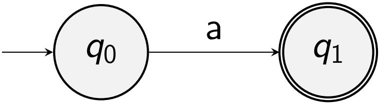

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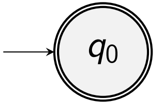

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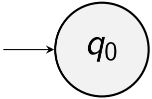

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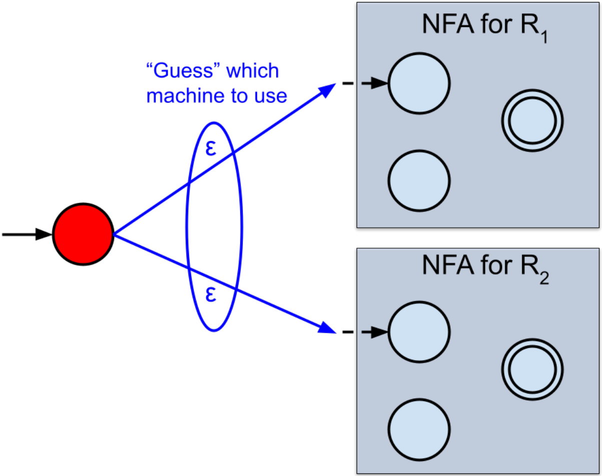

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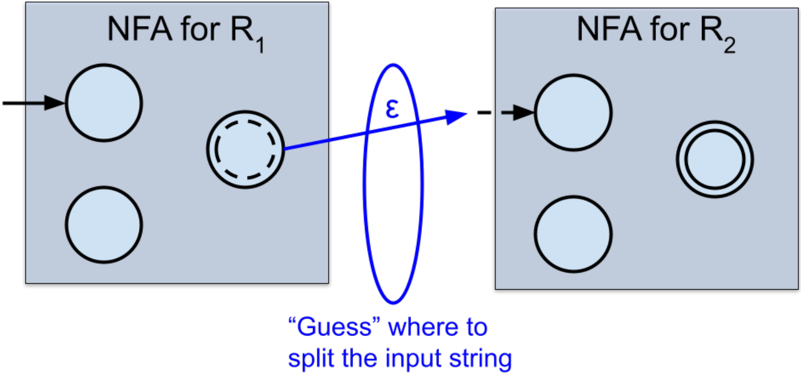

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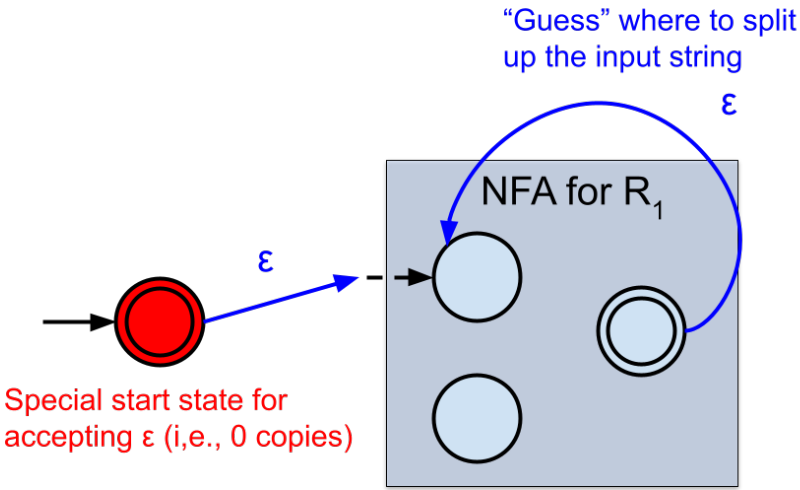

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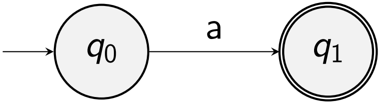

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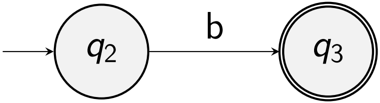

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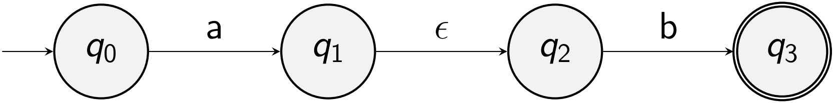

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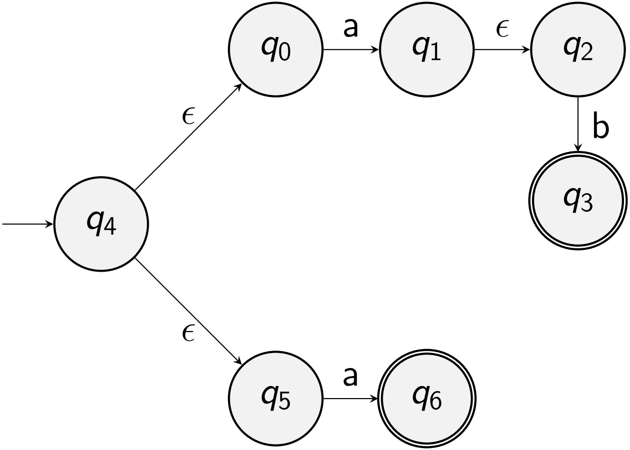

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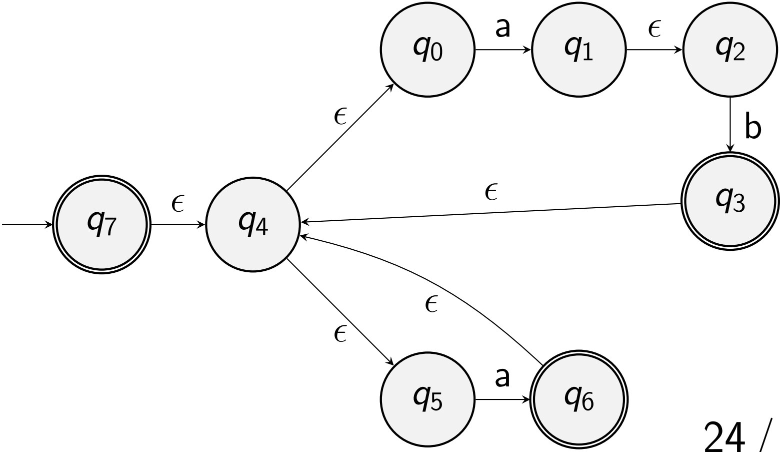

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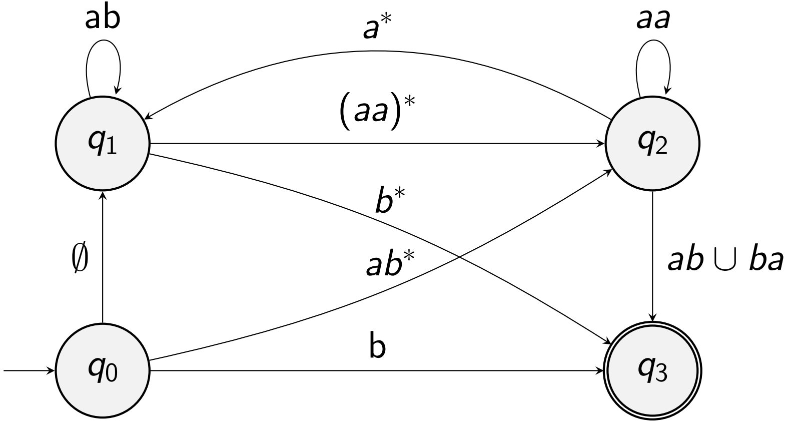

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- Except for the start and accept states, one arrow goes from every state to every other state, and also from each state to itself.

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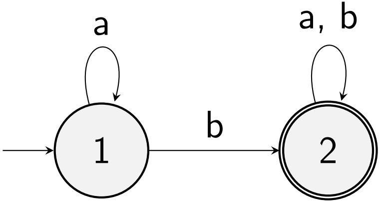
   - We can omit these when drawing state diagrams

DFA to GNFA

Starting DFA



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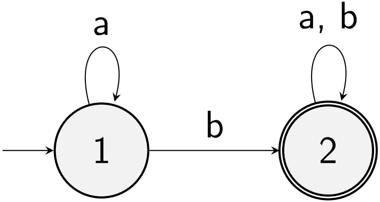
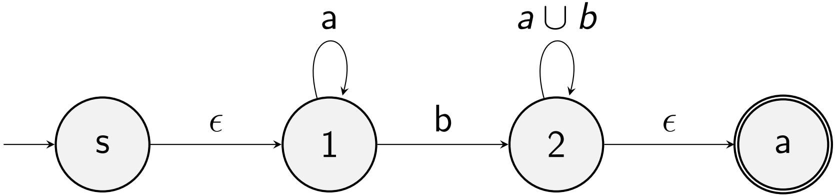
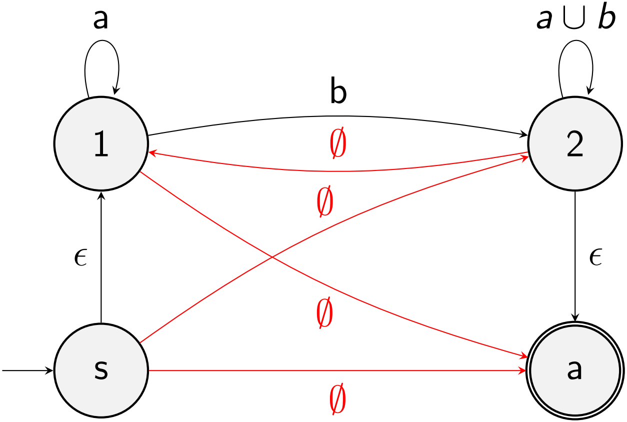


Starting GNFA



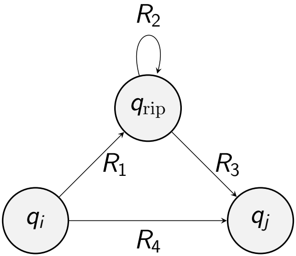
DFA to GNFA

Full starting GNFA



Ripping a state

How could we get from q_i to q_j?



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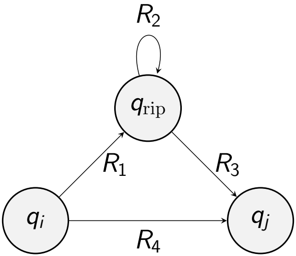

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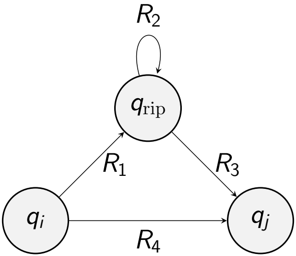

1. Go directly from q_i to q_j
2. Go through $q_{\rm rip}$

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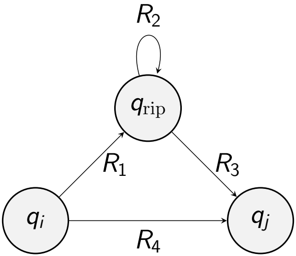

1. Go directly from q_i to q_j
2. Go through q_{rip}
   2.1 $q_i \to q_{\mathrm{rip}}$

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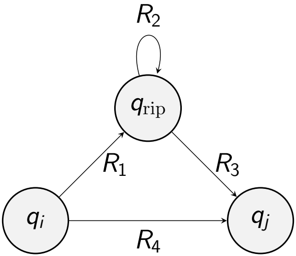

1. Go directly from q_i to q_j
2. Go through q_{rip}
   2.1 $q_i \to q_{\text{rip}}$
   2.2 $q_{\text{rip}} \to q_{\text{rip}}$ any number of times

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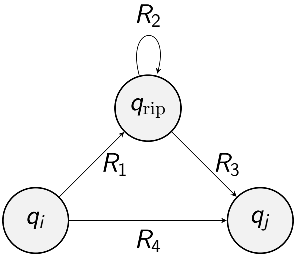

1. Go directly from q_i to q_j
2. Go through q_{rip}
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   2.2 $q_{\text{rip}} \to q_{\text{rip}}$ any number of times
   2.3 $q_{\text{rip}} \to q_j$

Ripping a state

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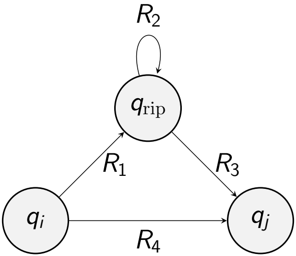

1. Go directly from q_i to q_j (R_4)
2. Go through q_{rip}
   2.1 $q_i \to q_{\text{rip}}$
   2.2 $q_{\text{rip}} \to q_{\text{rip}}$ any number of times
   2.3 $q_{\text{rip}} \to q_j$

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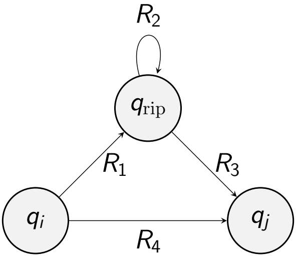

1. Go directly from q_i to q_j (R_4)
2. Go through q_{rip}
   2.1 $q_i \to q_{\text{rip}}$ (R_1)
   2.2 $q_{\text{rip}} \to q_{\text{rip}}$ any number of times
   2.3 $q_{\text{rip}} \to q_j$

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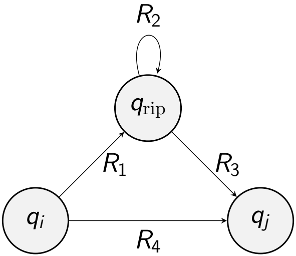
How could we get from q_i to q_j?



1. Go directly from q_i to q_j (R_4)
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   2.1 $q_i \to q_{\text{rip}}$ (R_1)
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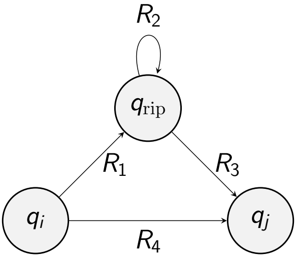

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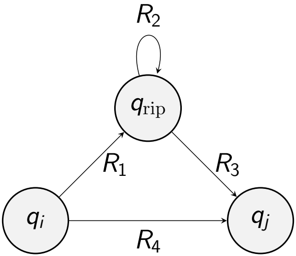

1. Go directly from q_i to q_j (R_4)
2. Go through q_{rip} ($R_1 \circ R_2^* \circ R_3$)
   - 2.1 $q_i \rightarrow q_{\mathrm{rip}}$ (R_1)
   - 2.2 $q_{\mathrm{rip}} \rightarrow q_{\mathrm{rip}}$ any number of times (R_2^*)
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Ripping a state

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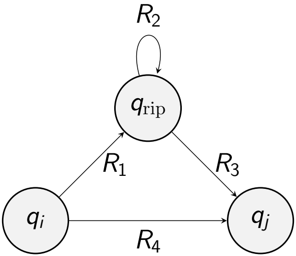

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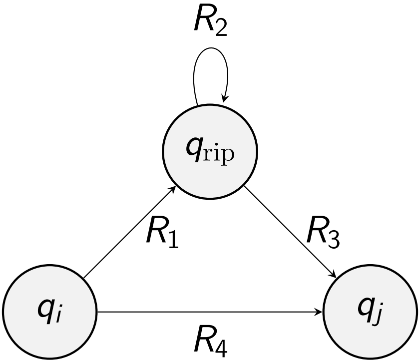
$R = (R_1 \circ R_2^* \circ R_3) \cup R_4$

Ripping a State

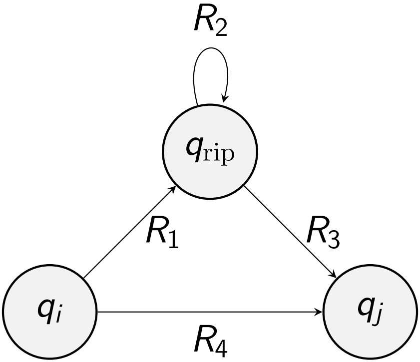
Ripping a State

Before

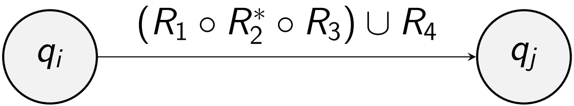


Ripping a State

Before

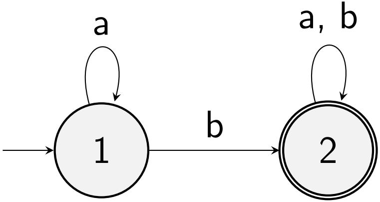


After



DFA to Regex

Starting DFA



DFA to Regex

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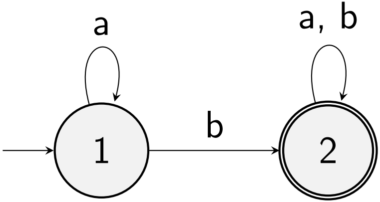
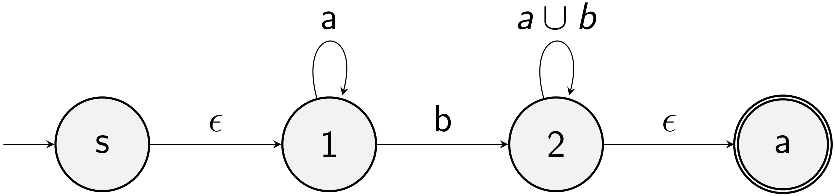
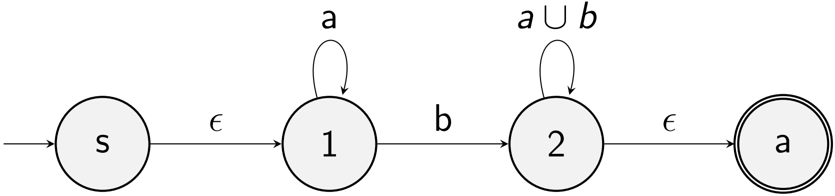


Starting GNFA

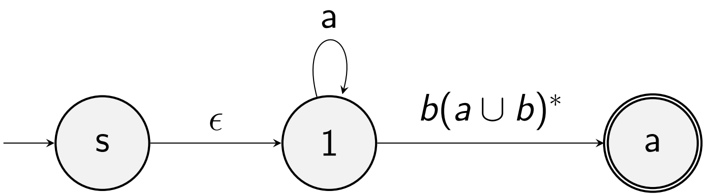


DFA to Regex

Starting GNFA

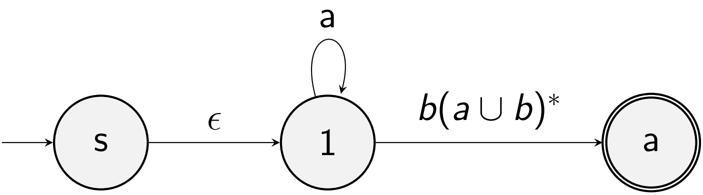


Rip State 2

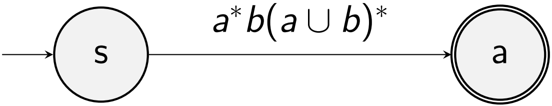


DFA to Regex

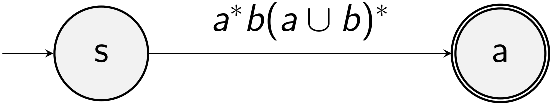
Rip State 2



Rip State 1



DFA to Regex



$$R = a^* b(a \cup B)^*$$

Regular expressions recap

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Regu

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Regular expressions recap

- Regular expressions are equivalent to NFAs
  - Which makes them equivalent to DFAs
- DFAs are equivalent to regular expressions
- A language is regular if and only if it is described by a regular expression
- To show a language is regular, can use a state machine or a regex

Regular expression closure proofs

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  - **Base case:** Show that closure holds for the three atomic regexes ($\emptyset, \epsilon, a$)

Regular expression closure proofs

- Regular expressions characterize the regular languages
- To show a language is regular, it is sometimes more convenient to use a regex than a state machine
- Sometimes, it is easier to write a closure proof using a regex
- **Blueprint:** Use an inductive proof
  - **Base case:** Show that closure holds for the three atomic regexes ($\emptyset, \epsilon, a$)
  - **Inductive case:** show that closure holds for t he three regular operations (union, concatenation, Kleene star)

EVERY-OTHER closure

Claim: If A is regular then $\text{EVERY-OTHER}(A)$ is regular

$$\text{EVERY-OTHER}(A) = \{w = \begin{array}{l} a_1 y_1 \dots a_n y_n | \\ a_1 \dots a_n \in A \\ y_i' s \text{ can be anything}\} \end{array}$$

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- Because A is regular, it is described by a regular expression R
- We will construct a regular expression R' that describes EVERY-OTHER(A)

EVERY-OTHER closure: base case

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