Turing Machine Variants

Arjun Chandrasekhar

Turing Completeness

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  2. Show that every language that can be recognized by a machine from the other model can be recognized a Turing machine

Stationary Turing Machine

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- The transition function is $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}$

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  1. If L is recognized by a normal TM, it can recognized by a stationary TM

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- There are **two** directions to this proof
  1. If L is recognized by a normal TM, it can recognized by a stationary TM
  2. If L is recognized by a stationary TM, it can be recognized by a TM

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- Let M be the TM that recognizes L

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- M **is** a stationary TM that simply chooses not to stay in place

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- Let M be the TM that recognizes L
- M **is** a stationary TM that simply chooses not to stay in place
- Thus, L can be recognized by a stationary TM

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- Create a machine M_2 behaves as M would, with one exception

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- Let M be the stationary TM that recognizes L
- **Technique:** create a normal TM that simulates M
- Create a machine M_2 behaves as M would, with one exception
- If M is supposed to stay in place, M_2 will move left and then move right before proceeding

2-hop Turing Machine

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  - What are the two directions?

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2-hop Turing Machine

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- Let M be the TM that recognizes L
- M **is** a 2-hop TM that chooses to only move one square at a time.

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- We will design a normal TM M_2 to simulate M

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- Let M be the 2-hop TM that recognizes L
- We will design a normal TM M_2 to simulate M
- M_2 operates as M would. If M tries to hop two spaces right or left, M_2 will perform the two hops over two consecutive steps

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  - What are the two directions?

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($\Rightarrow$) If L is recognized by a 1-tape TM, it can be recognized by a 2-tape TM

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- Let M be the 1-tape TM that recognizes L

2-tape Turing Machine

(⇒) If L is recognized by a 1-tape TM, it can be recognized by a 2-tape TM

- Let M be the 1-tape TM that recognizes L
- M **is** a 2-tape TM that ignores the second tape

2-tape Turing Machine

($\Leftarrow$) If L is recognized by a 2-tape TM, it can be recognized by a 1-tape TM

- Let M be the 2-tape TM that recognizes L

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- Let M be the 2-tape TM that recognizes L
- We will design a 1-tape TM called M_2 to recognize L

2-tape Turing Machine

($\Leftarrow$) If L is recognized by a 2-tape TM, it can be recognized by a 1-tape TM

- Let M be the 2-tape TM that recognizes L
- We will design a 1-tape TM called M_2 to recognize L
- M_2 will use its single tape to keep track of both of M's tapes.

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- Let M be the 2-tape TM that recognizes L
- We will design a 1-tape TM called M_2 to recognize L
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- At every step, M_2 simulates both tape heads of M

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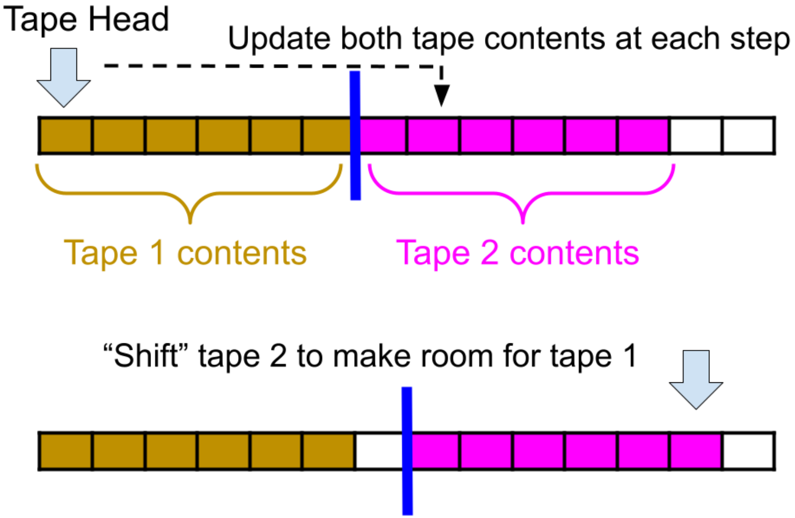
- Let M be the 2-tape TM that recognizes L
- We will design a 1-tape TM called M_2 to recognize L
- M_2 will use its single tape to keep track of both of M's tapes.
- At every step, M_2 simulates both tape heads of M
- If needed, M_2 can always push the second tape farther downstream to make more room for the first tape

2-tape Turing Machine



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  - There are many possible computation paths for the same string

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  - There are many possible computation paths for the same string
  - The machine accepts if at least one computation path accepts
- Let's show non-deterministic TMs are equivalent to deterministic TMs
  - What are the two directions?
  - Keep in mind that some computation paths may not halt

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($\Rightarrow$) If L is recognized by a deterministic TM, it can be recognized by a non-deterministic TM

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- Let M be the machine that recognizes L

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- Let M be the machine that recognizes L
- M **is** a non-determinstic TM that only has one computation path

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- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M

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- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M
- We run M, and try all possible computation paths.

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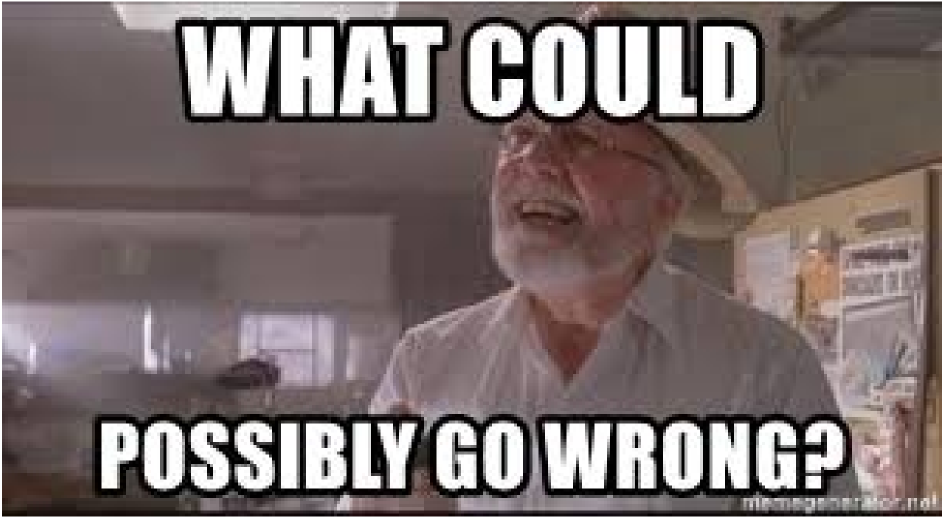
- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M
- We run M, and try all possible computation paths.
- If any computation path accepts, M_2 accepts, otherwise it rejects

Non-deterministic Turing Machine

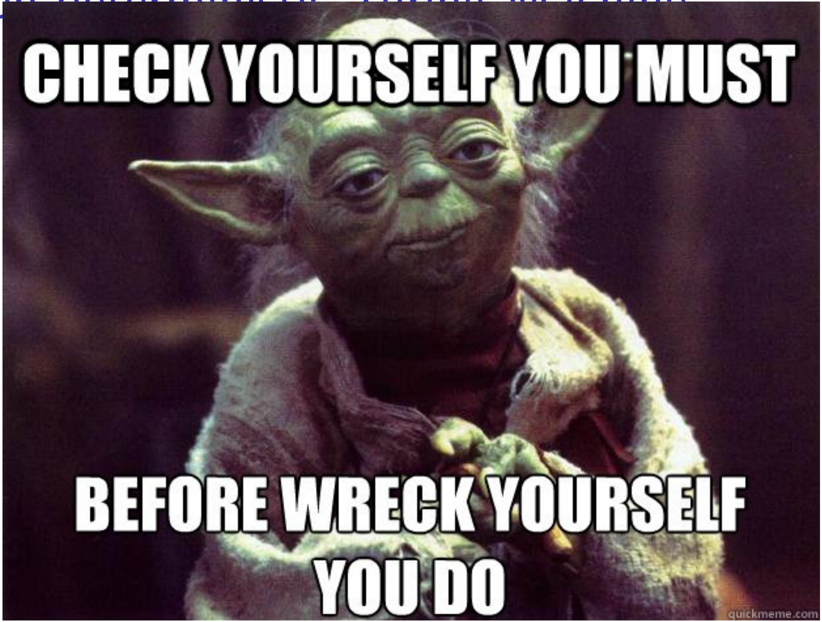


Non-deterministic Turing Machine

($\Leftarrow$) If L is recognized by a non-deterministic TM, it can be recognized by a deterministic TM

- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M
- ~~We run M, and try all possible computation paths.~~
  We might get stuck on a path that loops forever!
- If any computation path accepts, M_2 accepts, otherwise it rejects

Non-deterministic Turing Machine



Breadth-first search

Technique: We run M, and test out all possible computation paths in parallel

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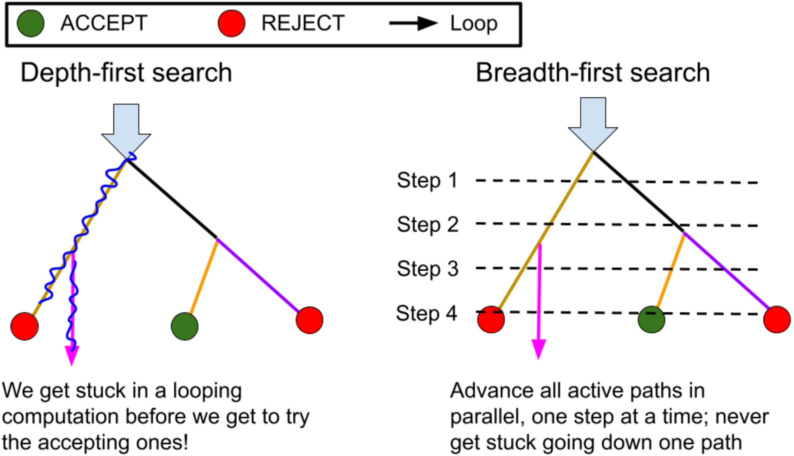
Breadth-first search



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- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M
- M_2 tries out all possible computation paths of M <u>in parallel</u>

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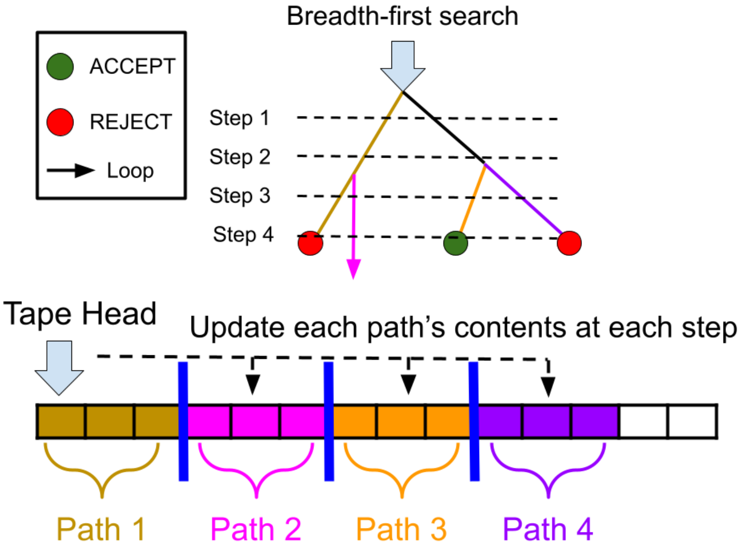
($\Leftarrow$) If L is recognized by a non-deterministic TM, it can be recognized by a deterministic TM

- Let M be the non-deterministic TM that recognizes L
- Design a deterministic TM called M_2 to simulate M
- M_2 tries out all possible computation paths of M in parallel
- If any computation path accepts ever, M_2 accepts, otherwise it rejects

Non-deterministic Turing Machine



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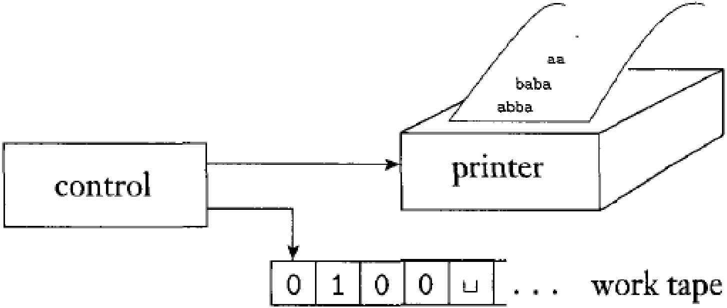

Recursively Enumerable Languages

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- We say E **enumerates** L if E prints out every string in L

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- If we give the enumerator infinite time, it will eventually print out every string in the language

Recursively Enumerable Languages

- Let L be a language, and let E be an enumerator
- We say E **enumerates** L if E prints out every string in L
- If we give the enumerator infinite time, it will eventually print out every string in the language
- **Def:** If L can be enumerated, we say L is **recursively enumerable (RE)**

Recursively Enumerable Languages

Proposition: The following language is recursively enumerable

$$L = \{p | p \in \mathbb{N}, p \text{ is prime}\}$$

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    1.1 Check if i is prime

Recursively Enumerable Languages

Proposition: The following language is recursively enumerable

$$L = \{p | p \in \mathbb{N}, p \text{ is prime}\}$$

1. For $i = 0, 1, 2, \ldots$
   1.1 Check if i is prime
   1.2 If i is prime, print it out

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Theorem: A language L is Turing-recognizable if and only if L is recursively enumerable.

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1. If L is Turing-recognizable, there is an enumerator E that enumerates L

Recursively Enumerable Languages

Theorem: A language L is Turing-recognizable if and only if L is recursively enumerable.

There are two directions:

1. If L is Turing-recognizable, there is an enumerator E that enumerates L
2. If L is RE, then some machine M can recognize L

Recursively Enumerable Languages

($\Rightarrow$) If L is recursively enumerable, L is Turing-recognizable

Recursively Enumerable Languages

($\Rightarrow$) If L is recursively enumerable, L is Turing-recognizable

- We know some machine E enumerates L

Recursively Enumerable Languages

($\Rightarrow$) If L is recursively enumerable, L is Turing-recognizable

- We know some machine E enumerates L
- Design a machine M to recognize L

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- On input w, do the following:

Recursively Enumerable Languages

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- We know some machine E enumerates L
- Design a machine M to recognize L
- On input w, do the following:
    1. Run E to enumerate L

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($\Rightarrow$) If L is recursively enumerable, L is Turing-recognizable

- We know some machine E enumerates L
- Design a machine M to recognize L
- On input w, do the following:
    1. Run E to enumerate L
    2. If E ever prints out w, then $w \in L$ so immediately accept

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($\Rightarrow$) If L is recursively enumerable, L is Turing-recognizable

- We know some machine E enumerates L
- Design a machine M to recognize L
- On input w, do the following:
  1. Run E to enumerate L
  2. If E ever prints out w, then $w \in L$ so immediately accept
  3. If E never prints out w, then $w \notin L$ and M will run forever (which is OK)

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Recursively Enumerable Languages

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- We know some machine M recognizes L
  - If $w \in L$, M accepts w

Recursively Enumerable Languages

(⇐) If L is Turing-recognizable, then L is recursively enumerable

- We know some machine M recognizes L
  - If $w \in L$, M accepts w
  - If $w \notin L$ then M rejects or loops

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- We design a machine E to enumerate L

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- We know some machine M recognizes L
  - If $w \in L$, M accepts w
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- We design a machine E to enumerate L
  1. Go through all $w \in \Sigma^*$ one at a time and run M on each w

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  2. If M accepts w, print out w.

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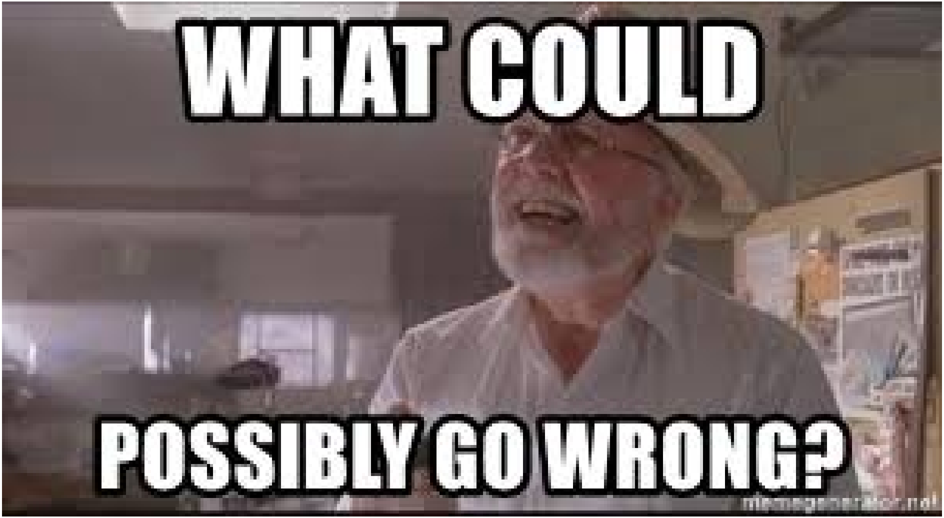
- We design a machine E to enumerate L
  1. Go through all $w \in \Sigma^*$ one at a time and run M on each w
  2. If M accepts w, print out w.
  3. After processing w, move on the the next string

Recursively Enumerable Languages



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- We know some machine M recognizes L
  - If $w \in L$, M accepts w
  - If $w \notin L$ then M rejects or loops
- We design a machine E to enumerate L
  1. Go through all $w \in \Sigma^*$ one at a time and run M on each w
  2. ~~If M accepts w, print out w.~~
     This may run forever!
  3. After processing w, move on the the next string

Dovetailing

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Technique: Run a machine M in parallel on all possible strings through dovetailing.

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- Let $w_1, w_2, \cdots \in \Sigma^*$
- Let $S(i, j)$ represent step i in M's computation on string w_j

Dovetailing

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  2. Run $S(1,2), S(2,1)$
  3. Run $S(1,3), S(2,2), S(3,1)$

Dovetailing

Technique: Run a machine M in parallel on all possible strings through dovetailing.

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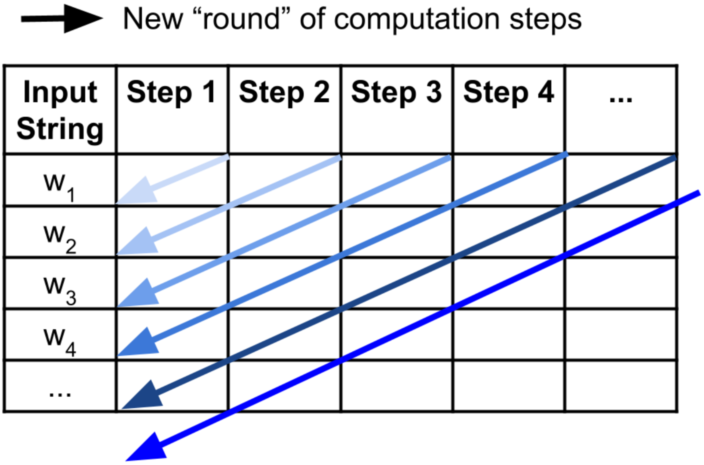
- Let $S(i, j)$ represent step i in M's computation on string w_j
  1. Run $S(1, 1)$
  2. Run $S(1, 2), S(2, 1)$
  3. Run $S(1, 3), S(2, 2), S(3, 1)$
  4. Run $S(1, 4), S(2, 3), S(3, 2), S(4, 1)$

Dovetailing

Technique: Run a machine M in parallel on all possible strings through dovetailing.

- Let $w_1, w_2, \cdots \in \Sigma^*$
- Let $S(i, j)$ represent step i in M's computation on string w_j
    1. Run $S(1, 1)$
    2. Run $S(1, 2), S(2, 1)$
    3. Run $S(1, 3), S(2, 2), S(3, 1)$
    4. Run $S(1, 4), S(2, 3), S(3, 2), S(4, 1)$
    5. …

Dovetailing



Recursively Enumerable Languages

($\Leftarrow$) If L is Turing-recognizable, then L is recursively enumerable

- We know some machine M recognizes L
  - If $w \in L$, M accepts w
  - If $w \notin L$ then M rejects or loops
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- We design a machine E to enumerate L
  1. Run M in parallel on all strings $w \in \Sigma^*$ (dovetailing)
  2. Whenever M accepts a string w, print out w (but keep running the other strings)